Determination of d and q Reactances of Permanent-Magnet Synchronous Motors Without Measurements of the Rotor Position

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***Abstract*—The interest in permanent-magnet synchronous motors (PMSMs) is increasing in a wide area of applications. Since most PMSMs will operate without a shaft sensor in the future, valuable information for experimental determination of machine parameters will be lost. In this paper, therefore, a method is presented where the induced EMF and the d-axis reactance are determined in a no-load test and the q-axis reactance is determined in a load test. The load angle δ is determined from the load test by means of a new analytical method. In this way, no separate measurement of the load angle is required. The method is especially suitable for line-start PMSMs which normally operate with negative d-axis current and, therefore, are not saturated in the d-axis flux paths. Moreover, the method is very simple to carry out for any laboratory technician, since the only tests that have to be made are standard tests which are made on standard induction motors on a regular basis.**

***Index Terms*—dq model, permanent-magnet machine, power factor, synchronous reactances.**

I. Introduction

THE interest for permanent-magnet synchronous motors (PMSMs) is currently increasing in a wide area of applications [1], [2] ranging from high-performance servo drives to line-start applications such as fans and pumps. There are mainly two reasons for this trend: first, the high efficiency and low rotor losses of the PMSM, and second, the falling prices of high-energy magnets.

Most three-phase PMSMs in operation today are high-performance drives with shaft sensors. In the future, however, it is the opinion of the authors that most PMSMs will operate position sensorless. This will be achieved by employing sensorless control algorithms [3] for variable-speed drives and, in the case of line-start applications [4], there is naturally no need for shaft sensors due to the synchronous operation with the mains.

As the shaft sensor is removed, valuable information for experimental determination of machine parameters is lost. It will be shown below that the expressions for the d- and q-axes reactances obtained from load tests are functions of the load angle δ, which can only be determined by means of some type of shaft sensor or another synchronous machine coupled to the shaft of the machine under investigation. In this paper, therefore, a method is presented where the induced EMF and the d-axis reactance are determined in a no-load test and the q-axis reactance is determined in a load test. It is the opinion of the authors that this simple test procedure, which requires no additional knowledge or equipment compared to standard induction motor tests, is easy to use for laboratory technicians who are used to induction motors. This makes the method especially interesting although it cannot guarantee high accuracy for highly saturated machines.

In Section II, an overview of methods to determine reactances experimentally is given. In Section III, the phasor diagram of the PMSM is presented and some basic relations are derived. In Section IV, it is shown how the load angle δ can be determined. In Section V, the test procedure is described and, finally, in Section VI, some conclusions are made.

II. Experiments to Determine Reactances

The test procedure described in Section V is basically a combination of a no-load test and a load test. These tests as such are not new, but the interpretation is new. In the literature, several other methods are described for the determination of the d- and q-axes reactances without the need for information of the rotor position in relation to the imposed quantities. One important distinction must, however, be made concerning the applicability of the methods. Reactances of, for instance, machines with surface-mounted magnets, i.e., without a damper winding characteristic, can easily be determined by means of various methods. A locked-rotor test with either a single-phase alternating voltage or a voltage step applied to the stator winding can be repeated for different rotor positions. A simple analysis of the results will then give comparably accurate results. It should also be noted that the choice of frequency, for the alternating voltage case, is not critical (50 or 60 Hz is good) and, in the voltage-step case, the magnitude of the voltage is not critical since only the initial slope is interesting. Methods to take saturation effects into account for these tests have also been presented; see, for instance, [5].

Machines with a damper winding characteristic, for instance, line-start permanent magnet synchronous motors (LSPMs),

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require a test procedure other than that for motors without a cage-like rotor. With a cage or a damper winding present, the synchronous reactance must be determined with a constant, or at least close to constant, magnetic flux in the rotor. If the frequency is too high, the flux will never penetrate the rotor due to the shielding effect of the cage. At high frequencies, therefore, only leakage reactance is observed, and the synchronous reactance is hidden. There are basically three purely experimental methods to determine the d- and q-axes reactances in this case:

1) by means of steady-state no-load and load tests with constant flux in the rotor;
2) by means of a test with a low-frequency alternating flux in the rotor;
3) by means of a test with a transient in the rotor flux.

Method 1) is very easy to carry out in any motor laboratory. This is a simple standard test usually made on standard induction motors, and all laboratory technicians are familiar with the test procedure.

Method 2) can be carried out, for instance, as a locked-rotor test. By supplying the winding with a low-frequency single-phase alternating voltage, a low-frequency magnetic flux is obtained in the rotor. By changing the rotor position in small steps for repeated consecutive measurements, the d- and q-axes reactances can be determined from measurements of voltage and current [5]. The choice of frequency is, however, not obvious, and a variable-frequency voltage source capable of providing very low frequencies is required.

Method 3) can also be carried out as a locked-rotor test. In this case, the stator winding is subjected to a voltage step. If the current transient is analyzed, the inductance for a certain position can be obtained. By repeating this measurement for different rotor positions, the d- and q-axes inductances can be determined. The analysis of the current transient is, however, difficult since different time constants have to be separated [5].

Both methods 2) and 3) make use of the low-frequency characteristics of the machine. With method 2), the analysis is made in the frequency domain and, with method 3), the analysis is made in the time domain. In both cases, saturation can be taken into account using a dc bias [5].

The low-frequency characteristics can also be investigated with more sophisticated methods, such as, for instance, by a combination of an energy perturbation approach with finite-element analyses [6], [7].

Also, other methods, including finite-element analyses in combination with measurements have been presented [8], [9].

All methods where finite-element analyses are required are, however, significantly more complex than methods 1)–3). Moreover, they require a detailed knowledge of the geometry of the machine. These methods are, therefore, not likely to be used in some kind of standard test procedure for laboratory technicians.

The methods presented in [10]–[12] require a precise measurement of the load angle, either by means of a stroboscope, a shaft sensor, or by applying another synchronous machine to the shaft. In [10], a double bridge is also suggested.

The most important problem for all methods is how to deal with saturation. As stated in [10], it is not possible to separate the induced magnet EMF E from the product of I_d and X_d at load. This is especially problematic since E may vary with load due to saturation. This problem is considered in, for instance, [9], but finite-element analyses are required.

In conclusion, it can be stated that an accurate reactance measurement, which is valid for all types of machines and all operating points, can only be made if all aspects of saturation are taken into account. The method presented in this paper focuses on simplicity, but it is still valid as long as d-axis saturation due to either d- or q-axes currents is not substantial.

III. Phasor Diagram for the Loaded PMSM

In Fig. 1, a phasor diagram of a loaded PMSM is shown. The figure is drawn for a typical case of operation for an LSPM. Due to the negative saliency, the motor operates with a comparably small negative d-axis current. This also implies that the d-axis flux paths are typically unsaturated. E is the phase value of the induced EMF from the magnets, U is the applied phase voltage, R_s is the stator resistance per phase, X_d and X_q are the reactances in the d and q directions, respectively, δ is the electrical angle between E and U defined positive for motor operation, ϕ is the angle between the stator current I and the voltage U, and I_d and I_q are the d- and q-axes components of the stator current.

From the phasor diagram, it is obvious that

$$X_d = \frac{U\,\cos(\delta) - E - R_s I_q}{I_d} \tag{1}$$

$$X_q = \frac{U\,\sin(\delta) + R_s I_d}{I_q} \tag{2}$$

$$I_d = I\,\sin(\phi - \delta) \tag{3}$$

$$I_q = I\,\cos(\phi - \delta). \tag{4}$$

If U, I, and the active input power are measured, the angle ϕ is known since

$$P = 3UI\,\cos(\phi) \tag{5}$$

assuming a three-phase circuit. The angle δ is, however, not known unless a separate measurement is made. This requires a stroboscope or some kind of shaft sensor. In the next section, it is shown how δ can be determined without a separate measurement of the load angle.

IV. Calculation of δ

Based on the assumption that E and X_d can be determined from a no-load test (see Section V-A) and that U, I, and ϕ are obtained from a load test, it will be shown that δ can be calculated from the phasor diagram. Since X_q is not yet known, an equation for δ must be found which does not contain X_q. As far as the authors have seen, this solution of the machine equations has not been presented before. It is this analytical solution which makes the test procedure easy. A simple pocket calculator is all that is required for the determination of δ.

Now, if the components in the q-axis direction are studied, (1) can be rewritten as

$$E + X_d I_d + R_s I_q = U\,\cos(\delta). \tag{6}$$

Inserting (3) and (4) into (6) yields

$$E + X_d I \sin(\phi - \delta) + R_s I \cos(\phi - \delta) = U \cos(\delta). \quad (7)$$

Since

$$\sin(\phi - \delta) = \sin(\phi)\cos(\delta) - \cos(\phi)\sin(\delta) \quad (8)$$
$$\cos(\phi - \delta) = \cos(\phi)\cos(\delta) + \sin(\phi)\sin(\delta) \quad (9)$$

(7) can be rewritten as

$$E = B \cos(\delta) + C \sin(\delta) \quad (10)$$

where

$$B = U - X_d I \sin(\phi) - R_s I \cos(\phi) \quad (11)$$

and

$$C = X_d I \cos(\phi) - R_s I \sin(\phi). \quad (12)$$

Now, (10) can be modified using

$$\sin(\delta) = \sqrt{(1 - \cos^2(\delta))}. \quad (13)$$

Rearranging and squaring on both sides of the equality yields

$$\left(\frac{E - B \cos(\delta)}{C}\right)^2 = 1 - \cos^2(\delta). \quad (14)$$

Using the substitution

$$y = \cos(\delta) \quad (15)$$

(14) can be rewritten as

$$y^2 - \frac{2EB}{B^2 + C^2}\, y + \frac{E^2 - C^2}{B^2 + C^2} = 0 \quad (16)$$

which is an ordinary second-order equation. Solving for y yields

$$y = \frac{1}{B^2 + C^2}\left[EB \pm \sqrt{B^2C^2 - C^2E^2 + C^4}\right] \quad (17)$$

where the minus sign normally gives the correct solution. As can be seen from Fig. 1, δ is normally larger than ϕ, which means that $y = \cos(\delta)$ should be smaller than the measured value of $\cos(\phi)$. In this way, it is easily checked which solution should be used.

Knowing δ, I_d and I_q can be obtained from (3) and (4), respectively. By insertion of I_d and I_q in (2), X_q is finally determined. In the next section, an overview of the tests is given.

V. Experimental Determination of X_d and X_q

The tests are carried out in two steps:

1) no-load test with variable voltage U;
2) load test at rated voltage U.

First, the no-load test is described, and then the load test is described.

A. No-Load Test with Variable Voltage U

By varying the voltage U, both E and X_d can be determined. The test can be made with different values of current by varying U. It must be kept in mind, however, that most motors operate with a small d-axis current in the rated operating point. LSPMs typically even operate with a small negative I_d. The purpose of this measurement is, therefore, to obtain values of E and X_d which are representative of a typical load case, i.e., when the d-axis flux paths are normally unsaturated. It must, therefore, be noted that for machines where the d-axis flux paths are heavily saturated in normal operation, the method is likely to fail.

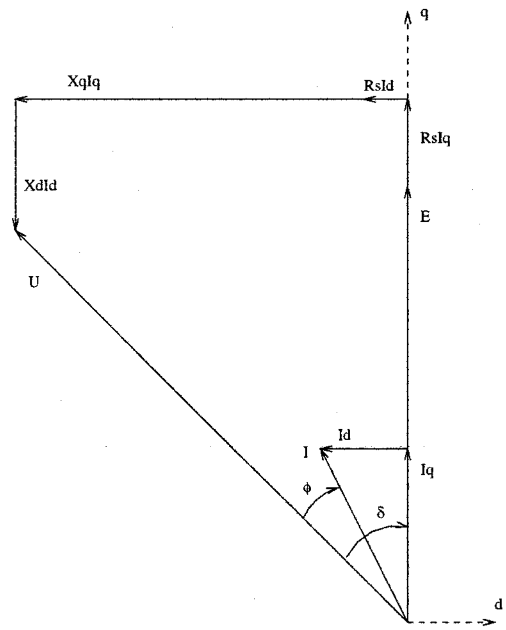

Fig. 1. Typical phasor diagram of a loaded PMSM fed directly from the mains.

From (1), it is seen that the resistive voltage drop results from the product of R_s and I_q. At no load, however, the current will have almost no q component and δ will be almost zero, as shown in Fig. 2. The load angle δ is slightly above zero due to friction and other losses.

As U is varied, I will also vary. At a certain point, a minimum of I is found. Now, U is approximately equal to E, and by reading the instruments E is determined.

Rewriting (1) with the assumptions that $\delta = 0$ and $I_q = 0$ yields

$$X_d = \frac{U - E}{I}. \quad (18)$$

This equation is, of course, not valid in the vicinity of the point $U = E$ since I_d then is zero and the only component of I is the small q-axis component required to overcome the losses. At a certain distance from this point, however, the linear relationship given in (18) is a good approximation. As E is constant, X_d can now be determined by reading the values of U and I and using (18). Since X_d can be determined for different values of I_d, a value of X_d can be found which is representative in a typical load case. In some cases, this may require some engineering experience from the actual type of machine.

B. Load Test at Rated Voltage U

Once the no-load test is made, E and X_d are determined. The motor shall now operate at rated voltage and shaft torque. By

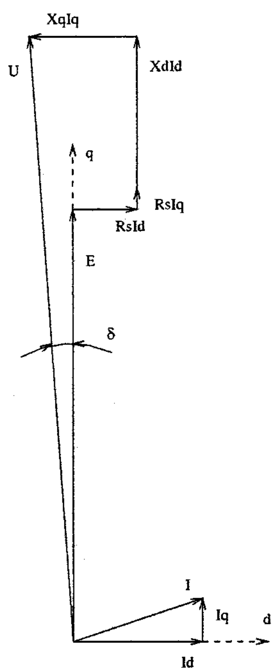

Fig. 2. Typical phasor diagram of a PMSM during no-load test. Note that the resistive voltage drops and the q-axis current are exaggerated for clarity.

measuring U, I, and P, the phase angle ϕ can be determined using (5). The next step is to determine δ as described in Section IV. Knowing δ, X_q can be determined. If the assumption that the d-axis flux paths are unsaturated is valid, then X_q can be determined as a function of load. However, as soon as the d-axis flux paths are saturated, either by I_d or I_q, the model will suffer from inaccuracies. In severe cases, there may not be any analytical solution for δ. The reason for this is that the values of E and X_d are not accurate. Typically, this should not be a problem due to the reasons stated above.

Another source of error is the presence of iron losses. These losses will add to $\cos(\phi)$. This implies that the value of $\cos(\phi)$ used in the equations is overestimated. This, in turn, normally results in an overestimation of X_q. Therefore, if accurate values are aimed at, the value of $\cos(\phi)$ at rated load should be decreased by an amount corresponding to the iron losses.

The iron losses are estimated from the no-load test by varying the voltage and measuring the input power. If the winding losses are subtracted from the measured power, only the iron losses and the friction losses are left. Since the friction losses are not influenced by the applied voltage, the iron losses can be determined assuming that they are proportional to the square of the applied voltage.

VI. Comparison of Analytical Calculations and Experiments

The method has been developed for LSPMs that were first tested without possibilities to measure the rotor position. To test the validity of the method, a special arrangement has been done to use a shaft sensor on prototypes.

The specifications of the tested motors are presented in Table I. There are 3 LSPMs and motor B is an integral motor

TABLE I
Motor Specificatons

Motor	A	B	C	D
type	LSPM	$\int M$	LSPM	LSPM
pole number	4	8	4	6
magnet shape	U	V	U	V
ext. diameter [mm]	250	254	210	210
length [mm]	150	110	115	115
induced voltage [V]	325	93.5	400	318

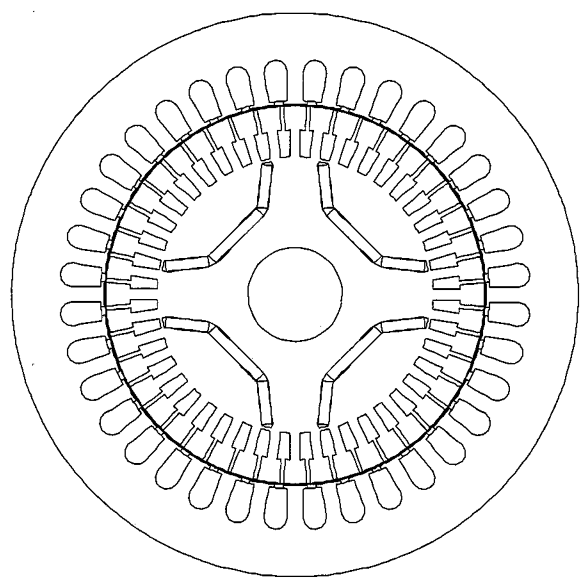

Fig. 3. Cross-sectional view of a four-pole LSPM.

($\int M$) [13]. The LSPMs have in common that they operate with a small negative d-axis current in the rated operating point. The integral motor will operate with zero d-axis current due to the inverter control. This means that the d-axis reactance is almost constant in the region where the motors normally operate. It also means that E is constant as long as I_q does not saturate the d-axis flux paths. However, at no load, it is unlikely that the LSPMs will operate with negative I_d, but it will be the case as soon as the load increases.

The LSPMs tested are three-phase motors with buried magnets in a U shape (motors A and C), as presented in Fig. 3. Motors D and B have magnets buried in a V shape. Fig. 4 shows the variation of the d- and q-axes reactances for motor A.

Table II shows the comparison between tests, finite-element method (FEM), and calculations with the presented method. Motor D data only allow one to determine the d-axis reactance from a no-load test and then to apply the method to calculate the q-axis reactance. The reactances for motors A and C are extracted from measurements by measuring the rotor position. For motor B, the reactances are estimated from measurements: a no-load test has been performed as well as a generator test to measure the d- and q-axes reactances [13].

However, the method shows its limits as it is not possible to calculate X_q for motor B by means of the method presented. The reason is that the tests of motor B were made before the measurement procedure of the method was derived. The load test was,

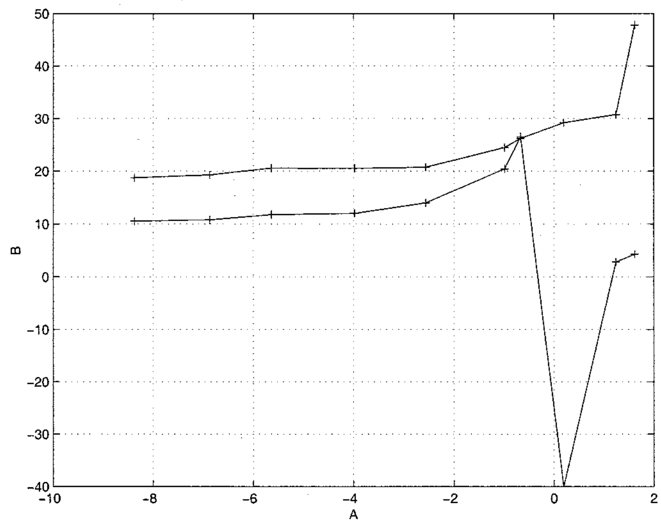

Fig. 4. X_d and X_q measurements for motor A.

TABLE II
TEST RESULTS, FEM, AND CALCULATIONS

Motor	A	B	C	D
$X_{d_{test}}$ [Ω]	5.2	(1.67)	10.8	*
$X_{q_{test}}$ [Ω]	9.9	(1.75)	19	*
$X_{d_{FEM}}$ [Ω]	5.5	1.55	*	*
$X_{q_{FEM}}$ [Ω]	11	1.72	*	*
$X_{d_{no-load}}$ [Ω]	5.7	1.67	10.2	8.85
$X_{q_{calc}}$ [Ω]	10	no	20.6	13.4

therefore, carried out with conditions that do not correspond to normal operation. The values of X_d and E from the no-load test could, therefore, not be used for determination of X_q since no analytical solution was obtained for the actual parameter set. With a new test, no problems of this kind are anticipated.

There is less than 10% error for motors A and C. However, the accuracy of the tested values is not known. The largest source of error is believed to originate from the determination of the d-axis reactance where the saturation is less than at rated load.

It is important to note that it is essential to have the correct values of R_s and E with respect to temperature. As the temperature increases, E decreases and R_s increases.

VII. CONCLUSION

A method to determine the d- and q-axes reactances of PMSMs has been presented. The method is new and can be used without a separate measurement of the load angle, a feature which will be important in the future since most PMSMs will not have any shaft sensors, especially the line-start motors. The main fact making it possible to determine the reactances without a separate measurement of the load angle is that the load angle is determined analytically from data obtained from a load test. The method is very simple and can be carried out by any laboratory technician, since the only tests that have to be made are standard tests which are made on standard induction motors on a regular basis. The method has been tested on various cases with different types of designs and the results show good agreements with theory. Comparisons with other methods and with finite-element analyses have also been presented.

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